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A new MRP estimate? A review of RBA discussion paper 2019-04

A report for Energy Networks Australia

August 2021

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Energy Networks Australia has asked HoustonKemp to examine the way in which a recent Reserve Bank of Australia (RBA) research discussion paper,

Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04,

produces estimates of the long-run market risk premium (MRP) for Australia.

Mathews constructs estimates of the MRP that use a new series of returns. In general, better estimates of the long-run MRP use:

- reliable data rather than unreliable data;
- longer time series rather than shorter time series; and
- broader indices rather than narrower indices.

Mathews estimates are based on broader indices than are existing estimates, but on a shorter time series than currently exists and on data that are less reliable than existing data. While we acknowledge that there can be trade-offs between the three criteria that we list, the problems with the reliability of the series of returns that Mathews constructs are sufficiently severe that we recommend that the Australian Energy Regulator (AER) place no weight on his estimates of the MRP.

Mathews computes a series of with-dividend returns from 1917 to 1979 to the largest 100 Australian stocks by market capitalisation, using dividends, prices and shares outstanding for each stock and for each quarter taken from various issues of the Sydney Stock Exchange *Official Gazette*, and then links these returns to series provided by Datastream from 1980 to 2019.

The problem that Mathews faces – and acknowledges that he faces – in using dividends, prices and shares outstanding alone to compute a series of returns is how to correctly handle capital changes. Capital changes include bonus issues, stock splits, reverse splits, the issuance of new shares and share buybacks. Mathews states that [1]

> The share price indices calculated are probably the least reliable of all the series due to the lack of information required to calculate an accurate divisor, a number used to deflate the index due to adjustments in the capital structure of included companies. To calculate this requires information about the terms of equity issuance, which is lacking from the RBA dataset: we can only infer issuance from a change in the number of shares outstanding.

The way in which Mathews addresses the problem of handling capital changes is to use changes in the number of shares outstanding as a signal that capital changes have taken place and a rudimentary classification scheme to adjust for the changes. He assume that: [2]

- if the number of shares that a firm has outstanding more than doubles over a quarter, the firm has carried out a stock split; and
- if the number of shares that a firm has outstanding does not more than double over a quarter, changes in the number of shares outstanding are the result of the firm buying back or issuing shares at a fair price.

This classification scheme, however, does not pick up bonus issues, which are and were common. Instead the classification scheme misclassifies bonus issues as issuances of new shares. Bonus issues are accompanied by stock price declines that are a purely mechanical consequence of the issues and in no way

[1] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

[2] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

harm investors. However, Mathews, through his classification scheme, feeds what appear to be the large negative returns associated with these stock price declines into his series of returns to the market. As a result, estimates of the long-run MRP that use his series of returns will be systematically downwardly biased.

The abstract of Mathews' paper states that:[3]

> the realised returns on equities has [sic] averaged about 4 percentage points above that on government bonds since 1917.

There are a number of reasons for this low value for an estimate of the long-run MRP. In declining order of importance, the major reasons for the low value that Mathews reports in the abstract to his paper are:

- the estimate uses the geometric means of samples of returns;
- the estimate ignores the impact of bonus issues; and
- the estimate is of the difference between the mean one-year return to the market and the mean one-year holding-period return to a 10-year bond rather than, as is conventional, a risk-free rate.

It is well known that the geometric mean of a series of returns to the market will lie substantially below the arithmetic mean of the series. So it is no surprise that the biggest factor contributing to the low value for the long-run MRP that Mathews reports in the abstract to his paper is his use of geometric means to compute the estimate.

Empirically, the one-year holding-period return to a 10-year bond lies, on average, above the 10-year yield. As a result, estimates of the long-run MRP that use one-year holding-period returns to a 10-year bond are lower than estimates that use 10-year yields. In contrast, the use of rational forecasts of one-year holding-period returns to a 10-year bond in place of 10-year yields will, on average, lead to higher estimates of the cost of equity for low-beta assets. As a theoretical matter, though, it is difficult to justify the use of rational forecasts of one-year holding-period returns to a 10-year bond to measure the risk-free rate. One-year holding-period returns are not risk-free and the Sharpe-Lintner variant of the capital asset pricing model, which the AER employs, requires the use of the return to a risk-free asset.

[3] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04.

1. Introduction

Energy Networks Australia (ENA) has asked HoustonKemp to examine the way in which a recent Reserve Bank of Australia (RBA) research discussion paper,

> Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04,

produces estimates of the long-run market risk premium (MRP) for Australia.

In particular, ENA has asked us to:

- explain why estimates of the MRP that Mathews generates differ from existing estimates;
- explain how the series that Mathews constructs differ from the Lamberton, Brailsford, Handley and Maheswaran, NERA and Dimson, Marsh and Staunton series;
- explain where Mathews uses methods or techniques not previously used in estimating the MRP for Australia;
- explain what impact using arithmetic or geometric mean returns will have on an estimate of the MRP and which measure one should use in estimating the return required on the assets of a regulated energy utility and why;
- explain whether the RBA paper has produced better estimates of the MRP for Australia than currently exist;
- make clear whether there are problems with the way in which Mathews constructs series of returns and whether these problems, if they exist, will lead the estimates of the MRP that he produces to be upwardly or downwardly biased; and
- identify which of the estimates of the long-run MRP for Australia that are available to the Australian Energy Regulator (AER) likely provide the most robust results, taking into account the regulator's Assessment Principles.

There are several differences between the way in which Mathews computes estimates of the long-run MRP and the way in which Brailsford, Handley and Maheswaran (2008, 2012), NERA (2013, 2015) and HoustonKemp (2017) compute estimates of the MRP. [4] The most important of these differences are:

- Mathews' use of a series of dividends, prices, shares outstanding and a rudimentary scheme for identifying the capital changes that firms carry out to construct a new series of returns to the market portfolio; and
- Mathews' use of one-year holding-period returns to 10-year bonds in place of 10-year bond yields to estimate the MRP.

[4] Brailsford, T., J. Handley and K. Maheswaran, *Re-examination of the historical equity risk premium in Australia*, Accounting and Finance 48, 2008, pp. 73-97.

Brailsford, T., J. Handley and K. Maheswaran, *The historical equity risk premium in Australia: Post-GFC and 128 years of data*, Accounting and Finance, 2012, pp. 237-247.

HoustonKemp, *A constructive review of the ERA's approach to the MRP: A report for Western Power*, June 2017.

NERA, *Market, size and value premiums: A report for the ENA*, June 2013.

NERA, *The market risk premium: Analysis in response to the AER's Draft Rate of Return Guidelines: A report for the Energy Networks Association*, October 2013.

NERA, *Further assessment of the historical MRP: Response to the AER's final decisions for the NSW and ACT electricity distributors: A report for ActewAGL Distribution, AGN, APA, AusNet Services, CitiPower, Energex, Ergon Energy, Jemena Electricity Networks, Powercor, SA Power Networks and United Energy*, June 2015.

Mathews' scheme for identifying the capital changes that firms carry out does not pick up bonus issues, which are and were common. The use of one-year holding-period returns to 10-year bonds in place of 10-year bond yields to estimate the MRP is unusual because these holding-period returns are not risk-free.

The rest of the report is organised as follows:

- section 2 provides an overview of Mathews' work;
- section 3 shows how a number of factors contribute to the low estimates of the long-run MRP that Mathews reports and assesses the relative importance of each factor;
- section 4 explains what impact Mathews' use of one-year holding-period returns to 10-year bonds in place of 10-year bond yields will have on estimates of the MRP;
- section 5 describes the impact that ignoring bonus issues will have on estimates of the MRP;
- section 6 examines the arguments for using arithmetic means and against using geometric means; and
- section 7 provides conclusions.

In addition:

- appendix A1 provides the terms of reference for this report.

2. Overview of Mathews' work

Mathews constructs estimates of the MRP that use a new series of returns. In general, better estimates of the long-run MRP use:

- reliable data rather than unreliable data;
- longer time series rather than shorter time series; and
- broader indices rather than narrower indices.

Mathews' estimates are based on broader indices than are existing estimates, but on a shorter time series than currently exists and on data that are less reliable than existing data. While we acknowledge that there can be trade-offs between the three criteria that we list, the problems with the reliability of the series of returns that Mathews' constructs are sufficiently severe that we recommend that the AER place no weight on his estimates of the MRP.

Existing estimates of the MRP for Australia that use long time series of returns typically use, before 1936, a price series for a commercial and industrial index, which runs from 1875 to 1936, that Lamberton (1958) supplies and a series of yields, which runs from 1882 to 1961, that Lamberton (1961) provides.[5] Mathews computes a series of with-dividend returns, from 1917 to 1979, to the largest 100 Australian stocks by market capitalisation, using dividends, prices and shares outstanding for each stock and for each quarter taken from various issues of the Sydney Stock Exchange (SSE) *Official Gazette*. He then links these returns to series provided by Datastream from 1980 to 2019.

The State Library of New South Wales reports that it has the SSE *Monthly stock and share list* from January 1914 to March 1937 while the State Library of Victoria reports that it has the *Australasian insurance and banking record* from 1877 to 1973.[6] Both sources carry dividends, prices and shares outstanding on a monthly basis and so it is difficult to see why Mathews chooses to use a time series that begins in 1917 and not one that begins earlier.

A problem in using dividends, prices and shares outstanding to compute a series of returns is how to correctly handle capital changes. Capital changes include bonus issues, stock splits, reverse splits, the issuance of new shares and share buybacks. To address the problem of handling capital changes, Mathews uses changes in the number of shares outstanding as a signal that capital changes have taken place and a rudimentary classification scheme to adjust for the changes. The classification scheme, however, does not pick up bonus issues, which are and were common. Instead the classification scheme misclassifies bonus issues as issuances of new shares. Bonus issues are accompanied by stock price declines that are a purely mechanical consequence of the issues and in no way harm investors. However, Mathews, through his classification scheme, feeds what appear to be the large negative returns associated with these stock price declines into his series of returns to the market. As a result, estimates of the MRP that use Mathews' series of returns will be systematically downwardly biased. We explain why in section 5 of this report.

As we make clear, better estimates of the long-run MRP use reliable rather than unreliable data, longer rather than shorter time series and broader rather than narrower indices. While the index that Mathews constructs is broad, the returns that he produces to the index are unreliable and the time series that he uses

[5] Lamberton, D, *Security prices and yields*, Sydney Stock Exchange Official Gazette, 14 July 1958.

Lamberton, D, *Ordinary share yields: A new statistical series*, Sydney Stock Exchange Official Gazette, 14 July 1961.

[6] https://collection.sl.nsw.gov.au/record/74VKMdp4XqkA

http://search.slv.vic.gov.au/primo-explore/fulldisplay?docid=SLV_VOYAGER1040344&context=L&vid=MAIN&lang=en_US&search_scope=Everything&adaptor=Local%20Search%20Engine&tab=default_tab&query=any,contains,australasian%20insurance%20and%20banking%20record

is shorter than it need be. It may be helpful then to examine how one might construct a reliable series of returns to a broad index over a long period in a cost effective way.

Existing estimates of the MRP for Australia that use long time series of returns typically use the price series for a commercial and industrial index that Lamberton provides, which again runs from 1875 to 1936, but do not use the price series for a financial index, which runs from 1875 to 1936, or the price series for a resources index, which runs from 1875 to 1910, that he also provides.[7] A glance at the latter two price series, that have lain largely unused since Lamberton constructed them over half a century ago, suggests that they provide information about some interesting episodes in Australian financial history that is not contained in the price series for the commercial and industrial index. For example:

- in 1887 the mining index rose by 133 per cent while the commercial and industrial index fell by 4 per cent;
- in 1890 the mining index rose by 117 per cent while the commercial and industrial index fell by 7 per cent; and
- in 1894 the financial index fell by 50 per cent while the commercial and industrial index fell by only 11 per cent.

Lamberton, who worked in the Research and Statistical Bureau of the SSE from 1949 to 1953, had access to the data necessary to correctly adjust for capital changes and so it is reasonable to view his three price series as reliable.[8] It would make sense then to construct a time series of returns to a broader market index, not from scratch, but by using his three price series together. Using the three series together would, of course, require time series of market weights for the three indices and producing these would be costly. Also, Lamberton's mining index runs only to 1910 and extending it to 1936 would be costly. Gathering the data for each stock necessary to correctly take account of all capital changes and so fix the reliability problems afflicting Mathews' series of returns, though, would surely be a massive task and so considerably more costly.

[7] Lamberton, D., *Security prices and yields*, Sydney Stock Exchange Official Gazette, 14 July 1958.

[8] Lodewijks, J, *Professor of foresight: An interview with Donald Lamberton*, Journal of Economic and Social Policy, 2007.

3. Differences between BHM, NERA and RBA estimates

There are a number of differences between the way in which Mathews computes estimates of the long-run MRP and the way in which Brailsford, Handley and Maheswaran (2008, 2012), NERA (2013, 2015) and HoustonKemp (2017) compute estimates of the MRP.[9] There are also differences between the way in which Brailsford, Handley and Maheswaran (BHM) compute estimates of the MRP and the way in which NERA and HoustonKemp compute estimates of the MRP. In this section we explain what these differences are and how they contribute to differences between estimates of the MRP that use the methods of BHM, HoustonKemp, Mathews and NERA.

Table 1: Factors contributing to gap between BHM, NERA and RBA estimates of the MRP

n	Estimate	MRP(n)	MRP(n) – MRP(n-1)
1	BHM data, theta = 0.65, 1883-2020, arithmetic mean, bond yields	6.36%	
2	NERA data, theta = 0.65, 1883-2020, arithmetic mean, bond yields	6.71%	0.35%
3	NERA data, theta = 0, 1883-2020, arithmetic mean, bond yields	6.45%	-0.26%
4	NERA data, theta = 0, 1917-2018, arithmetic mean, bond yields	6.12%	-0.33%
5	NERA data, theta = 0, 1917-2018, arithmetic mean, bond holding-period returns	5.74%	-0.38%
6	Lamberton cap. gains, RBA div. yields from 1917-1957, BHM-NERA data from 1958-2018, theta = 0, arithmetic mean, bond holding-period returns	5.68%	-0.06%
7	RBA capital gains, RBA div. yields from 1917-1957, BHM-NERA data from 1958-2018, theta = 0, arithmetic mean, bond holding-period returns	5.17%	-0.50%
8	RBA capital gains and div. yields from 1917-1979, BHM-NERA data from 1980-2018, theta = 0, arithmetic mean, bond holding-period returns	5.00%	-0.18%
9	RBA series from 1917-2019, theta = 0, arithmetic mean, bond holding-period returns	5.00%	0.00%
10	RBA series from 1917-2019, theta = 0, geometric mean, bond holding-period returns	3.70%	-1.30%

[9] Brailsford, T., J. Handley and K. Maheswaran, *Re-examination of the historical equity risk premium in Australia*, Accounting and Finance 48, 2008, pp. 73-97.

Brailsford, T., J. Handley and K. Maheswaran, *The historical equity risk premium in Australia: Post-GFC and 128 years of data*, Accounting and Finance, 2012, pp. 237-247.

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3.1 BHM estimates of the MRP

We begin by providing estimates of the long-run MRP that use BHM's data. BHM's two papers provide the details of how they construct their data and so here we give only a broad outline. Further, since HoustonKemp and NERA use either BHM's data or updates of their data from 1958 onwards, we describe only how BHM assemble data from 1883 to 1957.

BHM construct a series of returns using the price series for the commercial and industrial value-weighted index of stocks that Lamberton (1958) provides and the equally weighted series of yields on a broad cross-section of stocks that pay dividends that Lamberton (1961) provides. [10] Since the yields are equally weighted and exclude stocks that pay no dividends, BHM multiply the yields by 0.75, as a rule of thumb, before using them together with the price series to construct a series of with-dividend returns.

BHM compute an estimate of the MRP using both bill returns and yields on government bonds that either have a maturity of 10 years or, in earlier data, a maturity as close as possible to 10 years. BHM provide estimates of the MRP under the presumption that theta, defined to be the value placed by the market on imputation credits distributed that have a face value of one dollar, takes on the value of zero, 50 cents or one dollar.

Here we update BHM's estimates that use 10-year bond yields using data from 1883 to 2020 and an assumption that theta is 0.65.[11] We use this value for theta because the AER uses this value in recent determinations – not because we believe it to be the true value for theta. Row 1 of Table 1 shows that an estimate of the long-run MRP that uses these updated data and arithmetic means is 6.36 per cent per annum. This compares to an estimate of 6.2 per cent, to one decimal place, that the AER provides in its December 2020 *Rate of return annual update* that is based on updated BHM data from 1883 to 2018 and a value for theta of 0.65.

3.2 NERA estimates of the MRP

NERA (2013, 2015) uses original sources to examine whether the rule-of-thumb adjustment to Lamberton's yields of 0.75 that BHM employ is accurate.[12] It finds that the adjustment that BHM make is too large in early years, when dividend yields are high, and too small in later years, when dividend yields are low. So NERA uses, instead of the rule-of-thumb adjustment that BHM employ, adjustments that rely on the original sources that NERA examines. From 2016 onwards, Dimson, Marsh and Staunton also use the NERA adjustments in constructing returns from Lamberton's price and yield series in issues of the Credit Suisse *Global Investment Returns Sourcebook* and *Yearbook* that they co-author. [13] Using the series of returns that NERA constructs

[10] Lamberton, D., *Security prices and yields*, Sydney Stock Exchange Official Gazette, 14 July 1958.

Lamberton, D., *Ordinary share yields: A new statistical series*, Sydney Stock Exchange Official Gazette, 14 July 1961.

[11] To update BHM's data, we extract daily data (for days on which the market was open) for the All Ordinaries Index (AS30) and the All Ordinaries Accumulation Index (ASA30) from Bloomberg. Like BHM, we extract imputation credit yields for December of each year from the Australian Taxation Office and take the yields on 10-year Commonwealth Government bonds from the RBA. Also, like BHM, we compute the annual with-dividend return to the market portfolio in data from 1981 onwards as the percentage change from one year to the next in the average December level of the All Ordinaries Accumulation Index. To produce gross returns, we add to the with-dividend return 65 per cent of the credit return – that is, the ratio of the credits provided by the All Ordinaries within a year to the level of the index at the start of the year. Like BHM, we compute an estimate of the MRP by averaging the difference between each year's gross return and the yield on a 10-year Commonwealth Government bond at the end of each year.

[12] NERA, *Market, size and value premiums: A report for the ENA*, June 2013.

NERA, *The market risk premium: Analysis in response to the AER's Draft Rate of Return Guidelines: A report for the Energy Networks Association*, October 2013.

NERA, *Further assessment of the historical MRP: Response to the AER's final decisions for the NSW and ACT electricity distributors: A report for ActewAGL Distribution, AGN, APA, AusNet Services, CitiPower, Energex, Ergon Energy, Jemena Electricity Networks, Powercor, SA Power Networks and United Energy*, June 2015.

[13] Dimson, E., P. Marsh and M. Staunton, *Credit Suisse Global Investment Returns Sourcebook*, 2016.

Dimson, E., P. Marsh and M. Staunton, *Credit Suisse Global Investment Returns Yearbook*, 2017-2021.

from Lamberton's data, NERA reports estimates of the long-run MRP that are higher than their counterparts computed using BHM's data.

Here we update NERA's estimates using data from 1883 to 2020 and an assumption that theta is 0.65. Row 2 of Table 1 shows that an estimate of the MRP that uses these updated data and arithmetic means is 6.71 per cent per annum. This is 35 basis points higher than an estimate that uses BHM's data.

3.3 Estimates that presume that theta is zero

Mathews does not provide a value to imputation credits distributed to investors. In other words, he presumes that theta is zero, consistent with the idea that Australia is a small open economy. Row 3 of Table 1 shows that an estimate of the long-run MRP that uses NERA's data, updated to 2020, and arithmetic means and that sets theta to be zero is 6.45 per cent per annum. This is 26 basis points lower than an estimate that uses the same data and arithmetic means but sets theta to be 0.65.

3.4 Estimates that use a shorter time series

Mathews uses price data from Q1 1917 to Q1 2019 – and so return data from Q2 1917 to Q1 2019 – and so a shorter time series than existing studies employ. Again, the State Library of New South Wales reports that it has the SSE *Monthly stock and share list* from January 1914 to March 1937 while the State Library of Victoria reports that it has the *Australasian insurance and banking record* from 1877 to 1973 and both sources carry dividends, prices and shares outstanding on a monthly basis.[14] So it is difficult to see why Mathews chooses to use a shorter time series. In general, it is better to use a longer time series than a shorter time series to estimate the long-run MRP because longer time series will deliver more precise estimates of the premium.

Since some of our data are annual, for the time being we assess the impact of cutting the sample from 1883 to 2020 to 1917 to 2018 rather than to Q2 1917 to Q1 2019. As Row 4 of Table 1 shows, cutting the sample in this way produces an estimate of the MRP of 6.12 per cent, 33 basis points lower than an estimate that uses a longer time series.

3.5 Estimates that use bond holding-period returns

It is conventional to construct an estimate of the MRP as the difference between the mean return to the market and the risk-free rate. This is largely because the capital asset pricing model (CAPM) of Sharpe and Lintner requires the MRP to be computed in this way. Mathews chooses, however, to compute the MRP as the difference between the mean return to the market and the mean return to a one-year holding-period return to a 10-year bond – although the one-year holding-period return is not risk-free.

Row 5 of Table 1 shows that using his approximation for the one-year holding-period return to a 10-year bond in place of the 10-year bond yield produces an estimate of the MRP of 5.74 per cent, 38 basis points lower than an estimate that uses the 10-year bond yield. [15]

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[14] https://collection.sl.nsw.gov.au/record/74VKMdp4XqkA

http://search.slv.vic.gov.au/primo-explore/fulldisplay?docid=SLV_VOYAGER1040344&context=L&vid=MAIN&lang=en_US&search_scope=Everything&adaptor=Local%20Search%20Engine&tab=default_tab&query=any,contains,australasian%20insurance%20and%20banking%20record

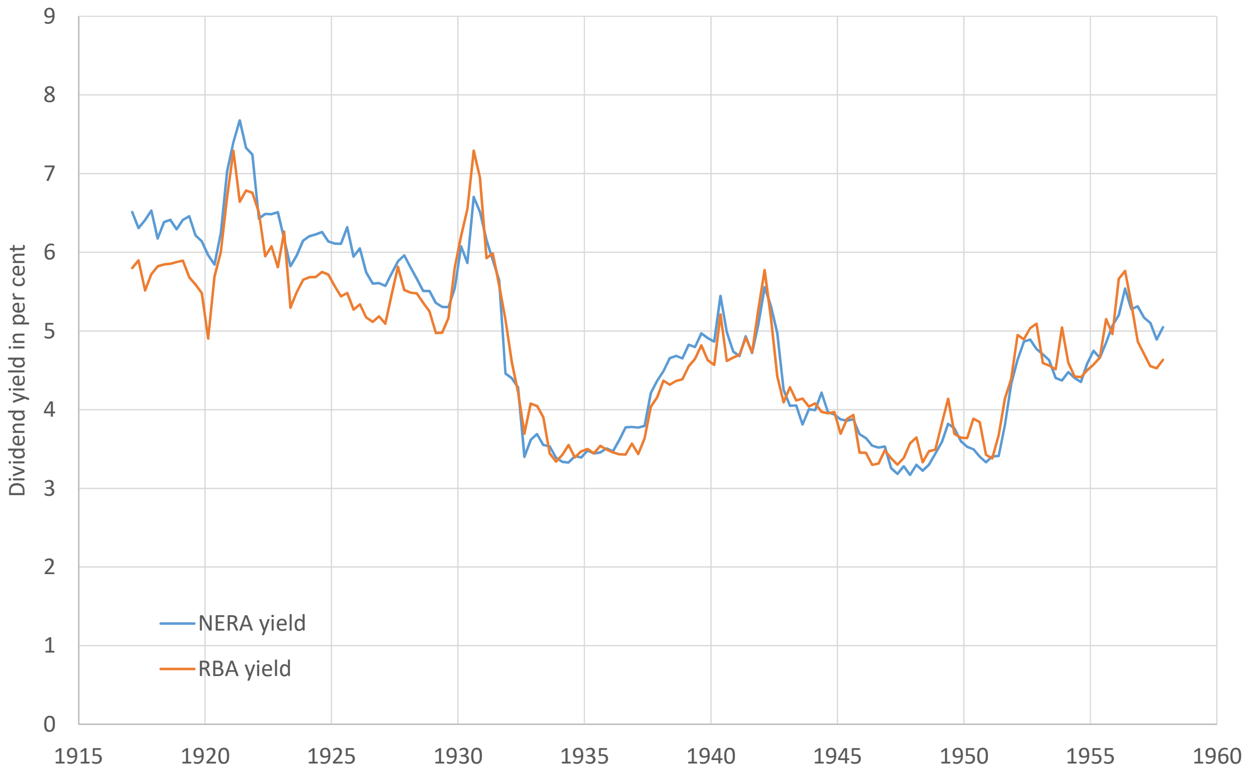
[15] Using the more accurate Shiller-Campbell-Schoenholtz approximation given by expression (3) in Section 4 to compute one-year holding-period returns to 10-year bonds rather than the approximation that Mathews employs produces an estimate of the MRP of 5.78 per cent. Thus of the 38 basis-point decline in an estimate of the MRP, four basis points are attributable to the use of Mathews less accurate approximation. For more details on the two approximations, see Section 4 of this report.

Shiller, R.J., J.Y. Campbell and K.L. Schoenholtz, *Forward rates and future policy: Interpreting the term structure of interest rates*, Brookings Papers on Economic Activity, 1983, pp. 173-223.

3.6 Estimates that use an alternative series of dividend yields

While the price series for the commercial and industrial index of stocks that Lamberton (1958) provides is value-weighted, the series of yields on a broad cross-section of stocks that pay no dividends that Lamberton (1961) provides is equally weighted.[16] Since the yields are equally weighted and exclude stocks that pay no dividends, BHM multiply the yields by 0.75, as a rule of thumb, before using them together with the price series to construct a series of with-dividend returns. Similarly, NERA uses a series of adjustments that rely on the original sources that they examine before using the yields together with the price series to construct a series of with-dividend returns.

Figure 1: Gap between NERA and RBA dividend yields



Mathews instead computes a series of yields directly from original sources. However, he does not compute a series of yields to Lamberton's commercial and industrial index but instead to a portfolio of stocks that includes financial and resources stocks. A basic proposition in finance is that, all else constant, the higher the coupons or dividends that an asset throws off, the lower will be the capital gains that the asset generates.[17] So combining the price series for Lamberton's commercial and industrial index with a series of yields on a portfolio that includes low-yield financial and resources stocks, without adjusting the series of yields to reflect differences between the composition of the index and portfolio, makes little sense. It makes

[16] Lamberton, D., *Security prices and yields*, Sydney Stock Exchange Official Gazette, 14 July 1958.

Lamberton, D., *Ordinary share yields: A new statistical series*, Sydney Stock Exchange Official Gazette, 14 July 1961.

[17] An example may help illustrate why this proposition makes sense. Consider a six-month US Treasury bill and an otherwise identical US Treasury note that has six months left to maturity and that will pay just one more coupon – at maturity. To avoid arbitrage, the returns to the two risk-free assets over the remainder of their lives must be identical. All of the bill's return over its life will come from capital gains while only a portion of the with-coupon return to the note will come from capital gains. It follows that the capital gain portion of the return to the note will be lower than the capital gain portion of the return to the bill.

little sense because it ignores the basic proposition that, all else constant, the lower the coupons or dividends that an asset throws off, the higher will be the capital gains that the asset generates.

Row 6 of Table 1 shows, though, that using the price series for Lamberton's commercial and industrial index together with Mathews' series of yields from 1917 to 1957 produces an estimate of the MRP of 5.68 per cent, only six basis points lower. Figure 1 shows why the impact of using Mathews' series of yields is so small. Only from 1917 to 1930 is there a significant gap between the yield on the commercial and industrial index – which here we label the NERA yield – and the yield on the portfolio that Mathews constructs that includes financial and resources stocks – which here we label the RBA yield.

3.7 Estimates that use Mathews' series of returns from 1917 to 1957

Mathews computes a series of with-dividend returns, from 1917 to 1979, to the largest 100 Australian stocks by market capitalisation using dividends, prices and shares outstanding for each stock and for each quarter taken from various issues of the SSE *Official Gazette*. The problem that Mathews faces in using these data to compute a series of returns is how to correctly handle capital changes. To address the problem, for each stock, he uses changes in the number of shares outstanding as a signal that capital changes have taken place and a rudimentary classification scheme to adjust for the changes. The classification scheme, however, does not pick up bonus issues, which are and were common and, as a result, estimates of the MRP that use Mathews' series of returns will be downwardly biased. Again, we explain why in section 5 of this report.

Row 7 of Table 1 shows that using the price series that Mathews constructs from 1917 to 1957 produces an estimate of the MRP of 5.17 per cent, 50 basis points (rounded to the nearest basis point) lower than an estimate that uses the price series that Lamberton provides and Mathews' dividend yields.

3.8 Estimates that use Mathews' series of returns from 1958 to 1979

Row 8 of Table 1 also shows that using the with-dividend series that Mathews constructs from 1958 to 1979 instead of the series that BHM construct and NERA employs from 1958 to 1979 produces an estimate of the MRP of 5.00 per cent, 18 basis points (rounded to the nearest basis point) lower than an estimate that uses the series that BHM and NERA employ.

3.9 Impact of other differences between Mathews and NERA data

There are a number of other differences between the data that Mathews uses and the data that NERA assembles and that we update and use. First, whereas Mathews uses data from Q2 1917 to Q1 2019, the sample of NERA's data that we use in Sections 3.4 to 3.8 run from Q1 1917 to Q4 2018.

Second, whereas Mathews uses, from 1987 onwards, either the ASX 100 or presumably, prior to its inception, an index of the largest 100 companies by market capitalisation, NERA, like BHM, from 1981 onwards, uses the broader All Ordinaries index.

Third, NERA, like BHM, computes the annual with-dividend return to the market, in data from 1981 onwards, as the percentage change from one year to the next in the average December level of the All Ordinaries Accumulation Index and it is unclear whether Mathews computes the return to an index of stocks in a similar manner or as the return to the index from the end of one year to the end of the next. Regardless, Row 9 of Table 1 shows that, together, these three differences have no impact on an estimate of the MRP.

3.10 Estimates that use geometric means

It is well known that estimates of the MRP that use geometric means lie far below estimates that use arithmetic means. Mathews also finds this to be true.

Row 10 of Table 1 shows that an estimate of the MRP that uses Mathews' data and geometric means is 3.70 per cent. This lies 130 basis points below an estimate of the MRP that uses his data and arithmetic means.

3.11 Summary

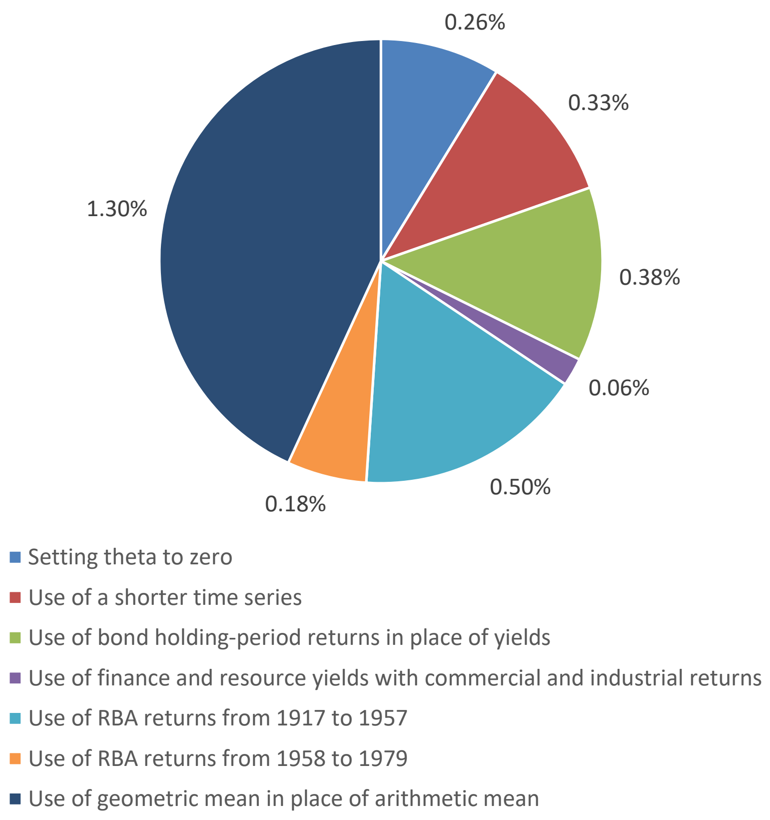
It is helpful to summarise these results using a pie chart as this will illustrate the relative importance of each difference between the ways in which Mathews and others go about computing an estimate of the MRP.

Figure 2 shows that by far the largest contributor to the difference between:

- Mathews' estimate of the MRP that uses the RBA series of returns from 1917 to 2019, theta = 0, geometric means and bond holding-period returns; and
- an estimate of the MRP computed using NERA's data, updated to 2020, theta = 0.65, arithmetic means and 10-year bond yields

is the use by Mathews of geometric means. The next largest contributor is the use of the returns that he assembles – the RBA returns. The third largest contributor is his use of one-year holding-period returns to 10-year bonds.

Figure 2: Factors contributing to gap between NERA and RBA estimates of the MRP



Since these are the three largest contributors, in section 4 we explain the impact that Mathews' use of one-year holding-period returns to 10-year bonds in place of 10-year bond yields has on estimates of the MRP and whether their use in estimating the cost of equity for a regulated energy utility using the CAPM of Sharpe and Lintner makes sense. In section 5, we describe the impact that ignoring bonus issues will have on estimates of the MRP and in section 6 we examine the arguments for using arithmetic means and against using geometric means in estimating the cost of equity for a regulated energy utility.

4. Impact of using bond holding-period returns

This section examines the impact of using one-year holding-period returns to 10-year bonds in place of 10-year bond yields on estimates of the MRP. We show that using one-year holding-period returns to 10-year bonds in place of 10-year bond yields will lead to lower estimates of the MRP.

On the other hand, the use of one-year holding-period returns to 10-year bonds in place of 10-year bond yields to estimate both the risk-free rate and the MRP will lead to higher estimates, on average through time, of the cost of equity for low-beta assets.

In the remainder of this section we:

- describe how Mathews computes the one-year holding-period returns to 10-year bonds;
- document the differences between the empirical behaviour of one-year holding-period returns to 10-year bonds and the empirical behaviour of 10-year bond yields; and
- assess the impact the use of one-year holding-period returns to 10-year bonds in place of 10-year bond yields will have on estimates of the MRP and estimates of the cost of equity.

4.1 Computing bond holding-period returns

Shiller, Campbell and Schoenholtz (1983) provide a simple method of computing approximate bond holding-period returns from bond yields.[18] Using their equation (3), an approximate holding-period return to a 10-year bond from year $t-1$ to year t will be:

$$D_t^{(10)} y_{t-1}^{(10)} - (D_t^{(10)} - 1) y_t^{(9)}, \tag{1}$$

where:

$D_t^{(10)}$ is the duration at t of a 10-year bond that trades at par; and

$y_t^{(i)}$ is the yield at t on an i-year bond.

The approximation (1) requires the yields on both 9-year bonds and 10-year bonds to compute the one-year holding-period returns to 10-year bonds. So, like Morningstar (2008), Mathews assumes, for simplicity, that the term structure at t is flat, thus enabling him to compute approximate one-year holding-period returns to 10-year bonds from just the yields to 10-year bonds. With this term-structure assumption and noting that[19, 20]

$$D_t^{(10)} = \frac{1 - \left(1 + y_t^{(10)}\right)^{-10}}{1 - \left(1 + y_t^{(10)}\right)^{-1}}, \tag{2}$$

expression (1) delivers the approximate holding-period return:

[18] Shiller, R.J., J.Y. Campbell and K.L. Schoenholtz, *Forward rates and future policy: Interpreting the term structure of interest rates*, Brookings Papers on Economic Activity, 1983, pp. 173-223.

[19] See equation (2) of Shiller, Campbell and Schoenholtz (1983).

Shiller, R.J., J.Y. Campbell and K.L. Schoenholtz, *Forward rates and future policy: Interpreting the term structure of interest rates*, Brookings Papers on Economic Activity, 1983, p 179.

[20] Morningstar, *Return calculation of US Treasury constant maturity indices*, Morningstar Methodology Paper, 2008.

$$\left(1-\left(1+y_t^{(10)}\right)^{-9}\right)\left(\frac{y_{t-1}^{(10)}}{y_t^{(10)}}-1\right)+y_{t-1}^{(10)} \tag{3}$$

Inspection of Mathews' code reveals that he derives an approximation for the one-year holding-period return to a 10-year bond differently and uses, instead of (3), the approximation:[21]

$$\left(1-\left(1+y_t^{(10)}\right)^{-10}\right)\left(\frac{y_{t-1}^{(10)}}{y_t^{(10)}}-1\right)+y_{t-1}^{(10)} \tag{4}$$

To see how well the approximation (3) works and to see whether the use by Mathews of the alternate approximation (4) leads him astray, we examine two simple examples of shifts in yields.

Table 2: Computing bond holding-period returns from yields

Year	Price	Yield	Return	SCS	RBA
			Panel A: Yields fall		
1	1.0000	10.00%			
2	1.0600	9.00%	16.00%	16.00%	16.42%
			Panel B: Yields rise		
1	1.0000	10.00%			
2	0.9446	11.00%	4.46%	4.46%	4.11%

Notes: The estimates use annual data from 1917 to 2018. The yields are on 10-year government bonds while the holding-period returns are one-year returns on 10-year government bonds. The column labelled SCS uses the Shiller-Campbell-Schoenhoeltz approximation (3) while the column labelled RBA uses the Mathews' approximation (4).

[21] The relevant R code contained in the file 'additional functions' reads as follows:

```
# helper function turning a bond yield into a total return index (this is used elsewhere too)
# assumes you sell the bond each year and buy a new one (therefore realising capital gain/loss)

udf_bondTR <- function(yield) {
yield <- yield / 100
yield <-
        ((lag(yield) * (1 - (1 + yield) ^ (-10)) / yield + 1 / (1 + yield) ^ 10) -
                1) + lag(yield) # (capital gain) + (coupon payments)
yield <- 1 + yield
yield[1] <- 1
yield <- cumprod(yield)

return(yield)

}
```

The file can be found at https://www.rba.gov.au/publications/rdp/2019/2019-04/supplementary-information.html

In Panel A of Table 2, the price of a bond that has a face value of one dollar, pays a single coupon of 10 per cent at the end of each year and trades at par at the end of year 1, rises over the following year. As a result, the yield of the bond, 10 per cent at the end of year 1, falls to 9 per cent at the end of year 2. The correctly computed holding-period return to the bond over the year is 16.00 per cent and, under the assumption that the term structure is flat at the end of year 2, the Shiller-Campbell-Schoenholtz (SCS) approximation given by (3) also produces, to two decimal places, a holding-period return of 16.00 per cent. Mathews' approximation – labelled RBA in Table 2 – produces, under the same assumption, a return of 16.42 per cent.

In Panel B of Table 2, the price of a bond that has a face value of one dollar, pays a single coupon of 10 per cent at the end of each year and that trades at par at the end of year 1, falls over the following year. As a result, the yield of the bond, 10 per cent at the end of year 1, rises to 11 per cent at the end of year 2. The correctly computed holding-period return to the bond over the year is 4.46 per cent and, under the assumption that the term structure is flat at the end of year 2, the SCS approximation given by (3) also produces, to two decimal places, a holding-period return of 4.46 per cent. Mathews' approximation produces, under the same assumption, a return of 4.11 per cent.

These examples show that Mathews' approximation will overstate the holding-period returns to bonds when yields fall and understate the returns when yields rise. However, we are primarily interested in mean returns, and while the impact of Mathews' use of the formula (4) on returns may be small but not negligible, the impact of the formula on mean returns will be negligible. If in Table 2 price falls and price rises are equally likely, then the impact of Mathews' use of the formula (4) will be to raise the mean return by just: [22]

$$0.5 \times (16.42\% - 16.00\%) + 0.5 \times (4.11\% - 4.46\%) = 0.04\% \qquad (5)$$

This analysis suggests that the two approximations that we examine, the SCS and RBA approximations, both work well – at least, when it comes to estimating mean holding-period returns.

4.2 Differences between bond holding-period returns and bond yields

To compute one-year holding-period returns to 10-year bonds we use the series that Brailsford, Handley and Maheswaran (2012) provide and that we update to 2020. [23] Although this updated series of yields runs from 1883 to 2020, we confine our analysis here to the period of 1917 to 2018 that Mathews examines.

Table 3: Differences between average bond yields and average holding-period returns

		Holding-period returns	
	Bond yield	SCS	RBA
Mean	6.11%	6.46%	6.50%
Standard deviation	3.13%	7.32%	7.76%

Notes: The estimates use annual data from 1917 to 2018. The yields are on 10-year government bonds while the holding-period returns are one-year returns on 10-year government bonds. The column labelled SCS uses the Shiller-Campbell-Schoenhoeltz approximation (3) while the column labelled RBA uses the Mathews' approximation (4).

Table 3 shows that the mean one-year holding-period return to a 10-year bond lies, on average:

[22] Rounded up to two decimal places.

[23] Brailsford, T., J. Handley and K. Maheswaran, *The historical equity risk premium in Australia: Post-GFC and 128 years of data*, Accounting and Finance, 2012, pp. 237-247.

- 35 basis points above the 10-year bond yield using the more accurate SCS approximation (3); and
- 39 basis points above using Mathews' approximation (4).

Table 3 also shows that one-year holding-period returns to 10-year bonds are not risk-free.

4.3 Impact of the use of bond holding-period returns on the cost of equity

The CAPM of Sharpe and Lintner predicts that the return required on an asset will be the sum of the risk-free rate and a risk premium that is the product of the asset's beta and the MRP. Alternatively, the CAPM predicts that the return required on an asset will be a weighted average of the risk-free rate and the return required on the market, where the weights on the risk-free rate and return required on the market depend on the asset's beta. Thus the CAPM predicts that:

$$\mathrm{E}(r_j) = r_f + \beta_j MRP = r_f + \beta_j \left(\mathrm{E}(r_m) - r_f\right) = (1-\beta_j)r_f + \beta_j \mathrm{E}(r_m) \tag{6}$$

The second formulation – that the return required on an asset will be a weighted average of the risk-free rate and the return required on the market – makes clear that there can only be one risk-free rate used in employing the model to estimate the return required on an asset. A user of the CAPM cannot use one measure of the risk-free rate to compute the return required on a risk-free asset and a second to compute the MRP, that is, the price of risk. So the AER cannot, for example, use the 10-year bond yield to compute the return required on a risk-free asset and one-year bond holding-period returns to estimate the MRP.

Empirically, the one-year holding-period return to a 10-year bond lies above the 10-year yield. As a result, using rational forecasts of one-year holding-period returns to a 10-year bond in place of 10-year yields will, on average, lead to lower estimates of the MRP but higher estimates of the cost of equity for low-beta assets. As a theoretical matter, though, it is difficult to justify the use of rational forecasts of one-year holding-period returns to a 10-year bond to measure the risk-free rate. One-year holding-period returns, as Table 3 shows, are not risk-free and the Sharpe-Lintner variant of the CAPM requires the use of the return to a risk-free asset.

5. Impact of ignoring bonus issues

This section examines the impact of ignoring bonus issues on estimates of the MRP. We show that ignoring bonus issues will cause estimates of the MRP to be downwardly biased.

Bonus issues, stock splits and reverse splits lead to changes in the number of shares a firm has outstanding with little or no change to the market value of the firm. Mathews correctly handles most stock splits and reverse splits are and were rare. However, he does not correctly handle bonus issues, and bonus issues are and were common.

In the remainder of this section we:

- describe how Mathews handles bonus issues, stock splits and reverse splits;
- explain what impact bonus issues will have on stock prices;
- examine the impact on stock prices of three large bonus issues carried out between 1919 and 1940; and
- describe the impact that ignoring bonus issues will have on estimates of the MRP.

5.1 RBA's method of handling capital changes

Mathews computes a series of with-dividend returns, from 1917 to 1979, to the largest 100 Australian stocks by market capitalisation using:

- dividends;
- prices; and
- shares outstanding

for each stock and for each quarter taken from various issues of the SSE *Official Gazette*.

The problem that Mathews faces – and acknowledges that he faces – in using these data to compute a series of returns is how to correctly handle capital changes. Mathews states that[24]

> The share price indices calculated are probably the least reliable of all the series due to the lack of information required to calculate an accurate divisor, a number used to deflate the index due to adjustments in the capital structure of included companies. To calculate this requires information about the terms of equity issuance, which is lacking from the RBA dataset: we can only infer issuance from a change in the number of shares outstanding.

The way in which Mathews addresses the problem is to assume that:[25]

- if the number of shares that a firm has outstanding more than doubles over a quarter, the firm has carried out a stock split; and
- if the number of shares that a firm has outstanding does not more than double over a quarter, changes in the number of shares outstanding are the result of the firm buying back or issuing shares at a fair price.

[24] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

[25] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

So Mathews computes the without-dividend return to a portfolio of the largest 100 stocks as:[26]

$$\frac{\sum_{\text{all } j} p_{jt} q_{jt}}{\sum_{\text{all } j \text{ s.t. } q_{jt} \le 2q_{jt-1}} p_{jt-1} q_{jt} + \sum_{\text{all } j \text{ s.t. } q_{jt} > 2q_{jt-1}} p_{jt-1} q_{jt-1}} - 1 \tag{7}$$

where:

p_{jt} is the price of stock *j* at time *t*; and

q_{jt} is the number of shares outstanding of stock *j* at time *t*.

Mathews' classification scheme will misclassify a number of capital changes. The scheme will misclassify:

- bonus issues and stock splits that a firm carries out that do not lead to a more than doubling of the number of shares of stock the firm has outstanding;
- reverse splits; and
- issues of new shares at a fair price that involve a more than doubling of the number of shares that a firm has outstanding.

It is likely that most stock splits that a firm executes involve a more than doubling of the number of shares the firm has outstanding and so the risk of Mathews misclassifying stock splits as share issuances is small. Mathews will misclassify reverse splits but reverse splits are rare and tend to involve distressed and so low-market-capitalisation firms. Consequently, the impact of Mathews misclassifying reverse splits will be small. It is also rare that firms issue new shares at a fair price and more than double the number of shares that they have outstanding. Moreover, any firms doing so are likely to be low-market-capitalisation firms and so the impact of Mathews misclassifying issuances of new shares as stock splits will also be small.

Bonus issues, though, are and were common and Mathews will misclassify virtually all of them. For this reason, we examine below the impact of bonus issues on stock prices.

5.2 Impact of bonus issues on stock prices

Suppose that:

- at time $t-1$, firm *j* has q_{jt-1} shares outstanding; and
- at time t, the firm carries out a bonus issue so that at *t* the firm has $q_{jt} > q_{jt-1}$ shares outstanding.

All else constant, the bonus issue should have no impact on the aggregate value of the firm's shares outstanding and so it must be the case that:

$$p_{jt} q_{jt} = p_{jt-1} q_{jt-1} \quad \text{and} \quad \frac{p_{jt}}{p_{jt-1}} - 1 = \frac{q_{jt-1}}{q_{jt}} - 1 \tag{8}$$

---

[26] We presume that the statement made by the RBA at the bottom of page 32 of its report that a split is presumed to have occurred only when $q_{jt} > 100 q_{jt-1}$ is a mistake.

Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

Thus, for example, a bonus issue whereby investors receive one additional share for each five shares of stock they already own will lead the price of the stock to fall by:

$$1-\frac{q_{jt-1}}{q_{jt}}=1-\frac{5}{6}=16.67\% \tag{9}$$

However, this price change does not represent real economic damage. The return to the firm's shareholders, all else constant, will be:

$$\frac{p_{jt}q_{jt}}{p_{jt-1}q_{jt-1}}-1=0\%, \tag{10}$$

that is, the firm's shareholders will be unaffected by the bonus issue. Ignoring the impact that bonus issues can have on stock prices, however, can lead estimates of the MRP to be downwardly biased. This is because the researcher will be misled into believing that the impact of bonus issues on stock prices harms shareholders.

In what follows, we provide three examples of large bonus issues carried out between 1919 and 1940 that led to large falls in prices but no corresponding harm to shareholders.

5.3 BHP bonus issue of 1919

Our first example is a large bonus issue carried out by BHP in 1919, shortly after the conclusion of the First World War. Although the issue was carried out after the war, it was planned before the war had ended, as the Age of 31 August 1918 makes clear. It states that:

> The half-yearly ordinary general meeting of the Broken Hill Proprietary Limited was held at the Equitable Building yesterday. The chairman of directors, Mr. Bowes Kelly, presided.
>
> The Chairman, when moving the adoption of the reports, recently reviewed in "The Age," said the directors greatly appreciated the manner in which the staff in Melbourne and London carried out the reconstruction scheme to a successful issue. Their labors were the more arduous because the registers contained the names of hundreds of Frenchman and Belgians who were in the fighting lines, and no effort was spared to afford all facilities in such cases. Of 8000 names on the registers, 5500 were on the London books. The Chairman recapitulated details of the reconstruction scheme, and concluded by stating the ultimate result would be that every shareholder holding 100 8/- shares to-day would receive in return 140 shares of £1 each. On the completion of the reconstruction the company would have an authorised capital of £3,000,000, of which £2,100,000, in 2,100,000 £1 shares, would be fully paid up – leaving a balance of £900,000 unissued, in 900,000 shares of £1 each.

The Age of 22 February 1919 provides more details and announces that:

> It has been decided to close the transfer books of the Broken Hill Proprietary Ltd. from 3 p.m., 5th March, until 18th March inclusive, to complete the issue of bonus shares and consolidation of shares under the scheme authorised by special resolutions confirmed 25th June, 1918. The bonus issue aggregates 600,000 shares, of 20/- each. These will be distributed at the rate of 40 shares for each 100 shares now held.

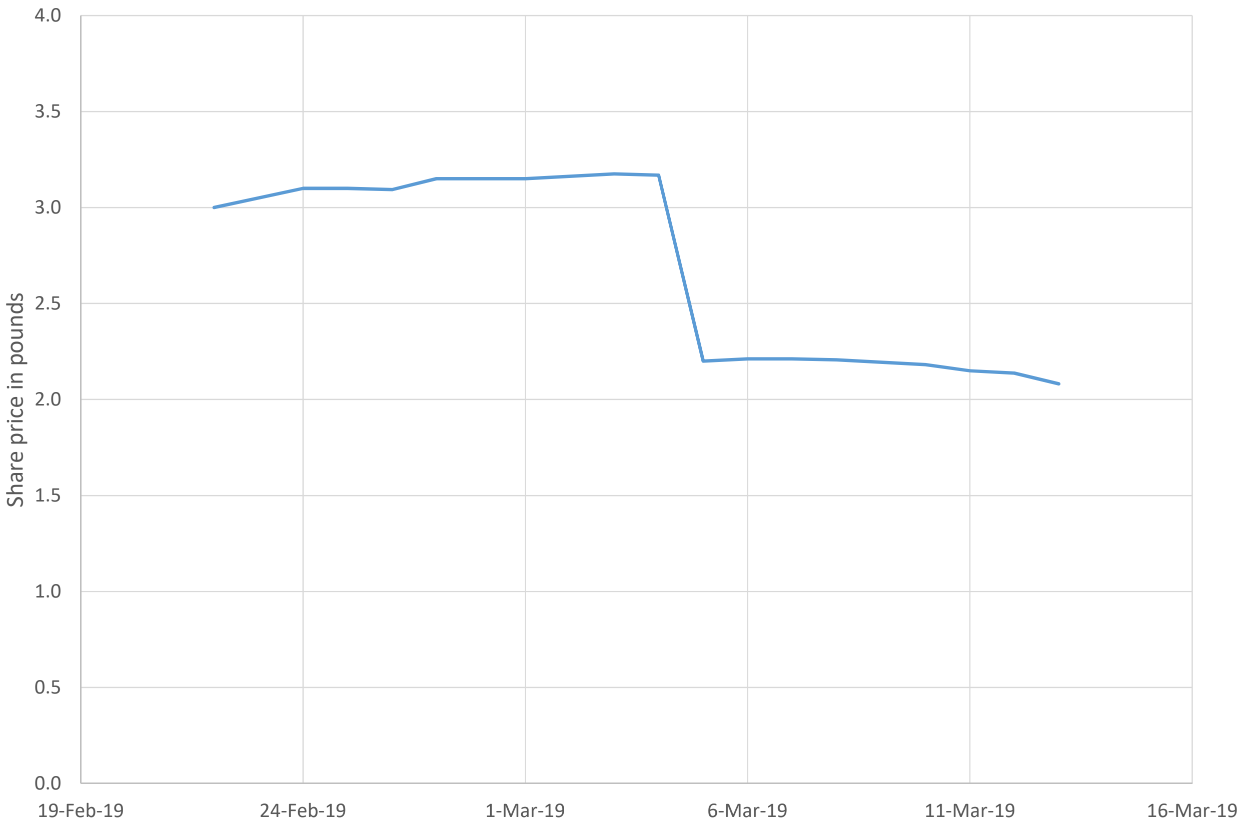
Equation (8) above indicates that the bonus issue should produce a fall in the price of a share of BHP of:

$$1-\frac{q_{jt-1}}{q_{jt}}=1-\frac{100}{140}=28.57\% \tag{11}$$

on the ex bonus date of 6 March 1919. The ex bonus date is the first day on which shares no longer trade cum bonus, that is, with the right to bonus shares. [27]

Figure 3 plots the price of a share of BHP from 22 February 1919 to 13 March 1919 using data taken from the Sydney Morning Herald. The prices are for the last sale of each day where a sale or sales are recorded and are the averages of the bid and ask prices where no sales are recorded. The figure shows clearly that the price of a share of BHP falls around the ex bonus date of 6 March 1919. Note that although the ex bonus date is 6 March 1919, the price falls on the day before because no cum-bonus prices are reported on the day – only ex-bonus prices.

Figure 3: BHP stock price around ex bonus date of 1919 bonus issue



The actual fall in the price of a share around the ex bonus date comes close to matching the theoretical prediction that it should fall by 28.57%. From 4 March 1919 to 5 March 1919, the price falls by 30.57%. While only 30.57% - 28.57% = 2.00% of this fall is not a mechanical consequence of the bonus issue, however, the RBA return series treats the entire fall in the price of a share of BHP as damaging to shareholders. In other words, Mathews feeds what appears to be the large negative return associated with the decline in the price of a share of BHP into his series of returns to the market. As a result of doing so for this and numerous other bonus issues, estimates of the long-run MRP that use his series of returns will be systematically downwardly biased.

[27] See the description of the code CB on the following page on the ASX web site: https://www.asx.com.au/research/announcements/status_notes.htm

5.4 CSR bonus issue of 1924

Our second example is a large bonus issue carried out by Colonial Sugar in 1924 that was prompted by the revesting of the Colonial Sugar Refining Company (Fiji and New Zealand), Limited, in the Colonial Sugar Refining Company, Limited. The Sydney Morning Herald of 7 March 1924 states that:

> At the forthcoming special meeting of shareholders of the Colonial Sugar Refining Company, after confirmation of the resolution for increasing the capital of the company, passed at the meeting held on February 21 last, the chairman will submit a resolution which will have the effect of distributing among shareholders one new share for every two shares now held.

The Sydney Morning Herald of 22 March 1924 provides further details and states that:

> A special meeting of shareholders of the Colonial Sugar Company, held yesterday, under the presidency of Mr. E. W. Knox, chairman of directors, confirmed the resolution passed at the meeting held on February 21, increasing the authorised capital of the company from £4,000,000 to £7,000,000, by the creation of 150,000 new shares of £20 each. Following the confirmation, a motion was submitted by the chairman, that in view of the revesting of the business of the Colonial Sugar Refining Company (Fiji and New Zealand), Limited, in the Colonial Sugar Refining Company, Limited, as owner of all the ordinary shares, the sum of £7,625,000 received from the liquidation of the former company, and being the balance of the proceeds of the assets of that company outside Australia, be capitalised and transferred to the share capital account of the Colonial Sugar Refining Company, and that this be effected by dividing the sum in the form of fully paid £20 shares in the capital of the company pro rata amongst the persons who on March 31 are registered holders of shares, and that the directors be directed to distribute among such persons 81,250 of the unissued £20 shares of the company credited as fully paid up, in the proportion of one of such share for every two shares in the capital of the company held by such persons.

Equation (8) above indicates that the bonus issue should produce a fall in the price of a share of CSR of:

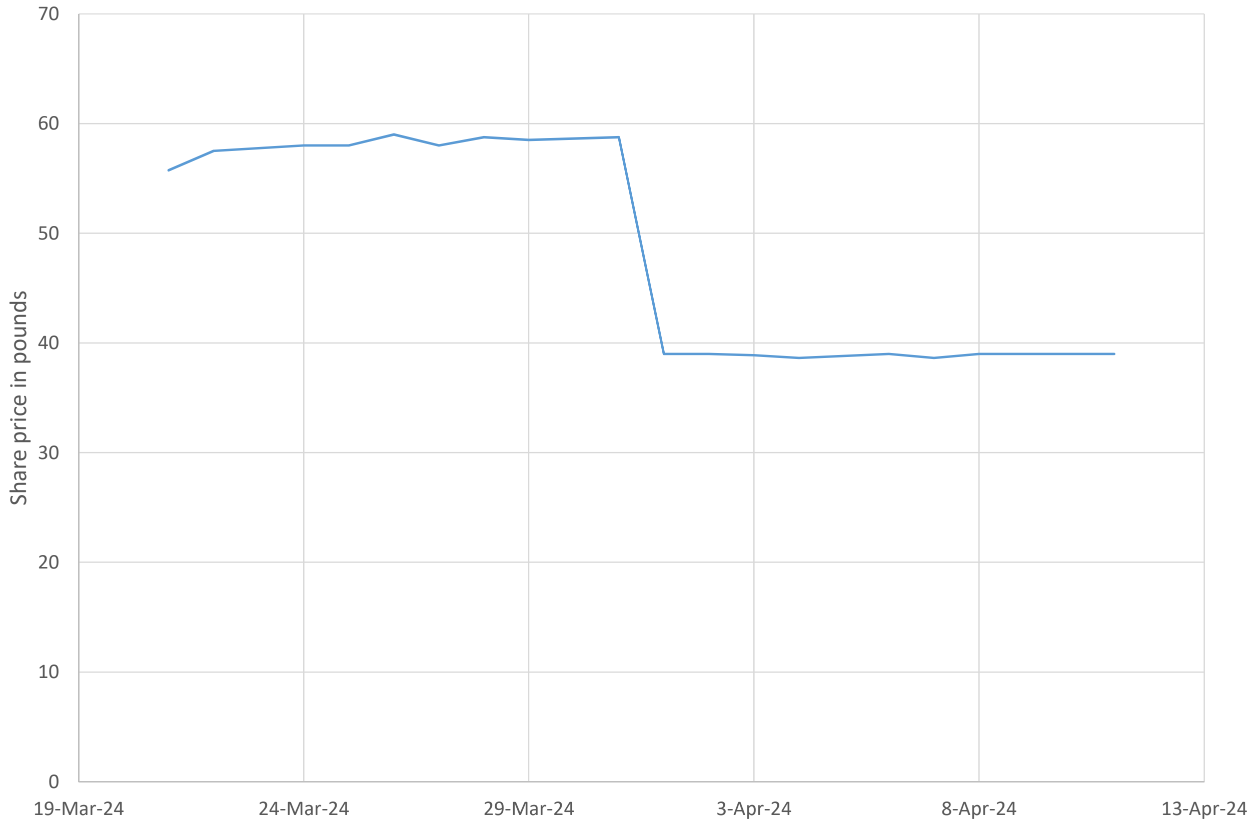
$$1-\frac{q_{jt-1}}{q_{jt}}=1-\frac{2}{3}=33.33\% \qquad (12)$$

on the ex bonus date of 1 April 1924.

Figure 4 plots the price of a share of CSR from 21 March 1924 to 11 April 1924 using data again taken from the Sydney Morning Herald and shows that the price of a share of CSR falls on the ex bonus date of 1 April 1924. The prices are once more for the last sale of each day where a sale or sales are recorded and are the averages of the bid and ask prices where no sales are recorded.

The actual fall in the price of a share on the ex bonus date comes very close to matching the theoretical prediction that it should fall by 33.33%. From 31 March 1924 to 1 April 1924, the price falls by 33.62%. While only 33.62% - 33.33% = 0.29% of this fall is not a mechanical consequence of the bonus issue, the RBA return series again treats the entire fall in the price of a share of CSR as damaging to shareholders.

Figure 4: CSR stock price around ex bonus date of 1924 bonus issue



5.5 BHP bonus issue of 1940

Our last example is the largest of the three bonus issues, carried out by BHP in the first year of the Second World War. The Daily News of 6 January 1940 providing details of the issue states that:

> New bonus shares to be issued shortly by the Broken Hill Pty., Ltd., will not be subject to Federal income tax nor to any State ordinary tax.
>
> The chairman, Mr. Harold Darling said this at a special meeting of shareholders.
>
> Because of technical reasons, the position under some of the State Acts imposing special taxes appeared to be somewhat obscure, he said.
>
> Transfer books will close on January 16, and the bonus shares – 64 to 100 – will be issued as soon after as possible.
>
> The meeting decided to modernise the company's articles of association and to increase the nominal capital from £15,000,000 to £25,000,000.

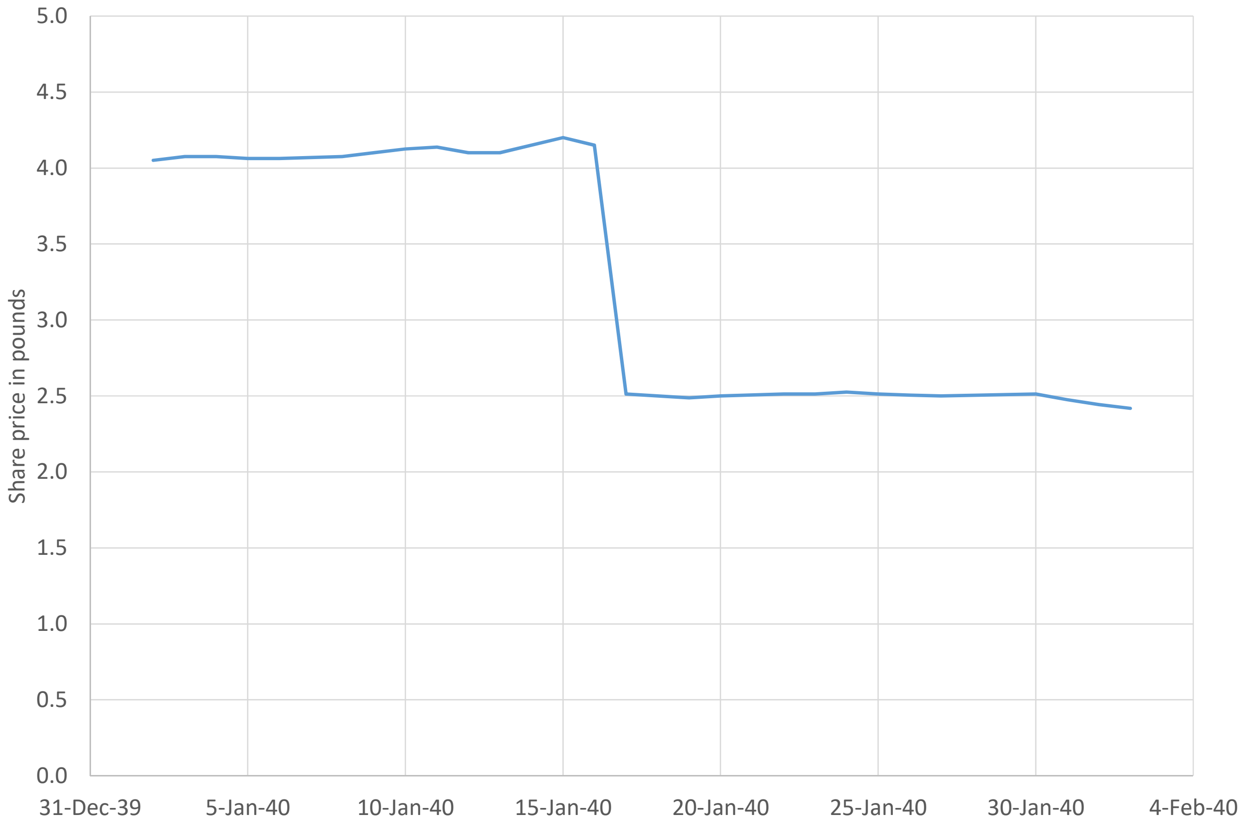
Equation (8) above indicates that the bonus issue should produce a fall in the price of a share of CSR of:

$$1-\frac{q_{jt-1}}{q_{jt}}=1-\frac{100}{164}=39.02\% \qquad (13)$$

on the ex bonus date of 17 Jan 1940.

Figure 5 plots the price of a share of BHP from 2 January 1940 to 2 February 1940 using data taken from the Sydney Morning Herald. Once more, the prices are for the last sale of each day where a sale or sales are recorded and are the averages of the bid and ask prices where no sales are recorded. The figure shows that the price of a share of BHP falls on the ex bonus date of 17 January 1940 and that the fall in price comes very close to matching the theoretical prediction that it should fall by 39.02%. From 16 January 1940 to 17 January 1940, the price falls by 39.46%. While only 39.46% - 39.02% = 0.44% of this fall is not a mechanical consequence of the bonus issue, the RBA return series again treats the entire fall in the price of a share of BHP as a decline in the value of existing shareholders' equity.

Figure 5: BHP stock price around ex bonus date of 1940 bonus issue



5.6 Impact of ignoring bonus issues on estimates of the MRP

As Mathews makes clear, he does not have the data necessary to properly account for the capital changes that firms carry out and so is unable to construct a reliable series of returns to the market. He can only use the time series of shares outstanding for each firm to infer what these capital changes might be. While it is likely that Mathews correctly handles most stock issuances and stock splits, Mathews ignores – and so does not correctly handle – bonus issues, which are and were common.

Bonus issues are accompanied by stock price declines that are a purely mechanical consequence of the issues and in no way harm investors. Mathews, through his classification scheme, though, feeds what appear to be the large negative returns associated with these stock price declines into his series of returns to the market. As a result, estimates of the MRP that use the series of returns that Mathews assembles will be systematically downwardly biased.

6. Impact of using geometric means

The arithmetic mean of a sample of returns when not compounded will be an unbiased estimator for the corresponding population mean return – so long as the mean return exists.[28] The arithmetic mean of a sample of returns when compounded over more than one period, however, will be an upwardly biased estimator for the corresponding population mean return over the same period.

In contrast, the geometric mean of a sample of returns when compounded over a sufficiently long period will be an unbiased estimator for the corresponding population mean return over the same period, but it will be a downwardly biased estimator for the corresponding population mean return over shorter periods.

Lally (2012) and NERA (2012) make clear that the AER never compounds an estimate of the weighted average cost of capital and so should completely avoid using geometric means.[29] Lally states in relation to the use of geometric means by the AER that:[30]

> The AER's belief that geometric averages are useful apparently arises from a belief that there is a compounding effect in their regulatory process (AER, 2012, Appendix A.2.1), and therefore the analysis of Blume (1974) and Jacquier et al (2003) applies. However, I do not think that there is any such compounding effect in regulatory situations and the absence of a compounding effect leads to a preference for the arithmetic mean over the geometric mean.
>
> If historical average returns are used, they should be arithmetic rather than geometric averages.

Like Lally, we recommend that the AER use only the arithmetic mean of a sample of returns to the market portfolio in excess of a measure of the risk-free rate to estimate the MRP and place no reliance on the geometric mean of the sample.

To examine the costs and benefits of using arithmetic and geometric means, define A to be the arithmetic mean of a sample of gross annual returns, that is, define:

$$A = \sum_{t=1}^{T} \frac{R(t)}{T}, \tag{14}$$

where:

$R(t)$ = one plus the rate of return to some asset from $t-1$ to t; and

T = the number of observations

and define G to be the geometric mean of a sample of gross annual returns, that is, define:

$$G = \left[\prod_{t=1}^{T} R(t)\right]^{1/T} \tag{15}$$

[28] There are random variables which have no means. The mathematical expectation of a Cauchy random variable, for example, does not exist. We assume that the expected values to which we refer exist.

[29] Lally, M., *The cost of equity and the market risk premium*, Victoria University of Wellington, 25 July 2012, pp. 31-32.

NERA, *The market risk premium: A report for CitiPower, Jemena, Powercor, SP AusNet and United Energy*, February 2012, pp. 3-12.

[30] ERA, *Final decision on proposed revisions to the access arrangement for the Dampier to Bunbury Natural Gas Pipeline 2016 – 2020 submitted by DBNGP (WA) Transmission Pty Limited: Appendix 4 Rate of return*, June 2016, pp. 118-119.

If the return to the asset is serially uncorrelated, the expected value of an estimate of the expected return to the asset over two years that uses the arithmetic mean will be:

$$\mathrm{E}(A^2) = [\mathrm{E}(A)]^2 + \mathrm{Var}(A) = \mathrm{E}(R(t)^2) + \mathrm{Var}(A) > \mathrm{E}(R(t)^2). \tag{16}$$

In words, the arithmetic mean when compounded over two years will be an upwardly biased estimator for the unconditional expected two-year return to the asset and the bias will reflect the imprecision with which the arithmetic mean estimates the population mean over one year.[31] The arithmetic mean when not compounded, on the other hand, will be an unbiased estimator for the unconditional expected one-year return to the asset – again, so long as it exists. That is, it will be true that:

$$\mathrm{E}(A) = \mathrm{E}(R(t)). \tag{17}$$

To gauge the bias associated with compounding the arithmetic mean of a sample of returns over more than two years and the bias that can be associated with either not compounding the geometric mean of a sample of returns or compounding the geometric mean over a sufficiently short period, it will be helpful to assume that:

$$\ln(R(t)) \sim NID(\mu, \sigma^2) \tag{18}$$

With this assumption, the unconditional expectation of the geometric mean of a sample of T returns compounded over n periods will be:

$$\mathrm{E}(G^n) = \mathrm{E}\left(\exp\left(\frac{n}{T}\sum_{t=1}^{T}\ln((R(t))\right)\right) = \exp\left(n\left(\mu + \frac{n}{2T}\sigma^2\right)\right) \tag{19}$$

while

$$\left(\mathrm{E}(R(t))\right)^n = \exp\left(n\left(\mu + \frac{1}{2}\sigma^2\right)\right) \tag{20}$$

From (19) and (20), the geometric mean of a sample of T returns compounded over $n < T$ periods will be a downwardly biased estimator for the corresponding population mean return over the same period.

To get an idea of the magnitude of the bias arising from compounding the arithmetic mean of a sample of returns and the bias from compounding the geometric mean of a sample of returns over a period that is shorter than the sample, we use NERA's data, that we update to 2020, (19), (20) and simulations.

Using the NERA data for the real return to the market from 1883 to 2020, we estimate μ to be 7.31 per cent and σ to be 16.47 per cent. Panel A of Table 4 shows that using these estimates, an estimate of the bias associated with compounding the arithmetic mean of a sample of 138 years of returns for five years will be four basis points per annum and an estimate of the bias associated with compounding the arithmetic mean of the sample for 10 years will be 10 basis points per annum.[32] NERA (2012) reports similar estimates using simulations and data from 1883 to 2011.[33] Thus the bias associated with compounding the arithmetic mean of a sample of 138 years of returns over 10 years or less is small. Panel A also shows that an estimate of the bias associated with not compounding the geometric mean of a sample of 138 years of returns will be minus 146 basis points per annum, an estimate of the bias associated with compounding the geometric mean of the sample for five years will be minus 142 basis points per annum and an estimate of the bias associated

[31] The unconditional expectation of a random variable is the mean of its marginal probability distribution. The conditional expectation of a random variable, on the other hand, is the mean of the probability distribution of a random variable conditional on some other variable or variables. Our focus in this section of the report is on unconditional expectations.

[32] $1.5454^{1/5} - 1.5424^{1/5} = 0.0004$ and $2.4002^{1/10} - 2.3789^{1/10} = 0.0010$

[33] NERA, *The market risk premium: A report for CitiPower, Jemena, Powercor, SP AusNet and United Energy*, February 2012, pp. 3-12.

with compounding the geometric mean of the sample for 10 years will be minus 136 basis points per annum.[34] Again, NERA (2012) reports similar estimates using simulations and data from 1883 to 2011.[35] The bias associated with compounding the geometric mean of a sample of 138 years of returns over 10 years or less is large.

Table 4: Bias in estimating mean multi-period returns and discount factors

	Horizon		
	1 year	5years	10 years
	Panel A: Mean multi-period returns in per cent		
Parameter value	9.05	54.24	137.89
Arithmetic mean	9.05	54.54	140.02
Geometric mean	7.59	44.48	109.77
	Panel B: Discount factors		
Parameter value	0.9170	0.6484	0.4204
Arithmetic mean	0.9172	0.6503	0.4251
Geometric mean	0.9296	0.6956	0.4862

Notes: The mean N-year return is $\exp\left(n\left(\mu+\frac{1}{2}\sigma^2\right)\right)$, *an estimate of the mean N-year return that uses the arithmetic mean is* A^N-1, *while an estimate of the mean N-year return that uses the geometric mean is* G^N-1. *The N-year discount factor is* $\exp\left(-n\left(\mu+\frac{1}{2}\sigma^2\right)\right)$, *an estimate of the N-year discount factor that uses the arithmetic mean is* A^{-N}, *while an estimate of the N-year discount factor that uses the geometric mean is* G^{-N}.

Entries for arithmetic means use the results of a simulation that employs 100,000 replications. Entries for geometric means use (19). The series of NERA real returns, updated to run from 1883 to 2020, is used to provide values for μ *and* σ. *The values the table uses are* $\mu = 7.31$ *per cent and* $\sigma = 16.47$ *per cent.*

Estimates of discount factors that use either the arithmetic or the geometric mean of a sample of returns will be upwardly biased and, all else constant, the bias associated with an estimate that uses the geometric mean will be larger. Panel B of Table 4 illustrates these facts.

A small literature examines how one might form an estimate of a mean multi-period return or discount factor by weighting estimates that use the arithmetic mean of a sample of returns and estimates that use the geometric mean.[36] From Panel A of Table 4, a weighted average that places a positive weight on both estimates of a mean multi-period return should be unbiased if one chooses the weights correctly. Panel A

[34] $1.0905-1.0759=0.0146, 1.5424^{1/5}-1.4448^{1/5}=0.0142$ and $2.3789^{1/10}-2.0977^{1/10}=0.0136$

[35] NERA, *The market risk premium: A report for CitiPower, Jemena, Powercor, SP AusNet and United Energy*, February 2012, pp. 3-12.

[36] See, for example, Cooper (1996).

Cooper, I., *Arithmetic versus geometric mean estimators: Setting discount rates for capital budgeting*, European Financial Management, 1996, pp. 157-167.

makes clear that while the weights will depend on the horizon, for horizons of 10 years or less, the weight placed on an estimate that uses the geometric mean of a sample of returns will be small. From Panel B of Table 4, a weighted average that places a positive weight on an estimate of a discount factor that uses the arithmetic mean of a sample of returns and a negative weight on an estimate that uses the geometric mean should be unbiased, if one chooses the weights correctly. Again, the weights will depend on the horizon and the weight placed on an estimate that uses the geometric mean of a sample of returns will be small.

The message from Table 4 for a regulator is that bias can arise if the regulator compounds an estimate of the allowed rate of return. Using weighted averages of mean multi-period returns that use the arithmetic mean of a sample of returns and the geometric mean of the sample can deliver unbiased estimates, but identifying the weights that these averages use requires the regulator make clear over what horizon it has compounded an estimate of the allowed rate of return.

Again, there is no evidence, however, that the AER ever compounds an estimate of the allowed rate of return. Although revenue must be forecast for each of the several years of the typical regulatory period, at no stage, aside from in making minor adjustments to the regulated asset base and to the evolution of prices, is the allowed rate of return compounded over more than one year. Thus an allowed rate of return that is based solely on the arithmetic mean of a sample of annual returns to the market portfolio in excess of a risk-free rate will – so long as the other components of the allowed rate of return have been correctly computed and ignoring minor adjustments – produce an unbiased estimate of the revenue that the market requires the utility earn in any single year of a regulatory period.

7. Conclusion

Energy Networks Australia has asked HoustonKemp to examine the way in which a recent Reserve Bank of Australia (RBA) research discussion paper,

Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04,

produces estimates of the long-run market risk premium (MRP) for Australia.

Mathews constructs estimates of the MRP that use a new series of returns. In general, better estimates of the long-run MRP use:

- reliable data rather than unreliable data;
- longer time series rather than shorter time series; and
- broader indices rather than narrower indices.

Mathews estimates are based on broader indices than are existing estimates, but on a shorter time series than currently exists and on data that are less reliable than existing data. While we acknowledge that there can be trade-offs between the three criteria that we list, the problems with the reliability of the series of returns that Mathews constructs are sufficiently severe that we recommend that the Australian Energy Regulator (AER) place no weight on his estimates of the MRP.

Mathews computes a series of with-dividend returns from 1917 to 1979 to the largest 100 Australian stocks by market capitalisation, using dividends, prices and shares outstanding for each stock and for each quarter taken from various issues of the Sydney Stock Exchange *Official Gazette*, and then links these returns to series provided by Datastream from 1980 to 2019.

The problem that Mathews faces – and acknowledges that he faces – in using dividends, prices and shares outstanding alone to compute a series of returns is how to correctly handle capital changes. Capital changes include bonus issues, stock splits, reverse splits, the issuance of new shares and share buybacks. Mathews states that [37]

> The share price indices calculated are probably the least reliable of all the series due to the lack of information required to calculate an accurate divisor, a number used to deflate the index due to adjustments in the capital structure of included companies. To calculate this requires information about the terms of equity issuance, which is lacking from the RBA dataset: we can only infer issuance from a change in the number of shares outstanding.

The way in which Mathews addresses the problem of handling capital changes is to use changes in the number of shares outstanding as a signal that capital changes have taken place and a rudimentary classification scheme to adjust for the changes. He assume that: [38]

- if the number of shares that a firm has outstanding more than doubles over a quarter, the firm has carried out a stock split; and
- if the number of shares that a firm has outstanding does not more than double over a quarter, changes in the number of shares outstanding are the result of the firm buying back or issuing shares at a fair price.

This classification scheme, however, does not pick up bonus issues, which are and were common. Instead the classification scheme misclassifies bonus issues as issuances of new shares. Bonus issues are accompanied by stock price declines that are a purely mechanical consequence of the issues and in no way

[37] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

[38] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04, p 32.

harm investors. However, Mathews, through his classification scheme, feeds what appear to be the large negative returns associated with these stock price declines into his series of returns to the market. As a result, estimates of the long-run MRP that use his series of returns will be systematically downwardly biased.

The abstract of Mathews' paper states that:[39]

> the realised returns on equities has [sic] averaged about 4 percentage points above that on government bonds since 1917.

There are a number of reasons for this low value for an estimate of the long-run MRP. In declining order of importance, the major reasons for the low value that Mathews reports in the abstract to his paper are:

- the estimate uses the geometric means of samples of returns;
- the estimate ignores the impact of bonus issues; and
- the estimate is of the difference between the mean one-year return to the market and the mean one-year holding-period return to a 10-year bond rather than, as is conventional, a risk-free rate.

It is well known that the geometric mean of a series of returns to the market will lie substantially below the arithmetic mean of the series. So it is no surprise that the biggest factor contributing to the low value for the long-run MRP that Mathews reports in the abstract to his paper is his use of geometric means to compute the estimate.

Empirically, the one-year holding-period return to a 10-year bond lies, on average, above the 10-year yield. As a result, estimates of the long-run MRP that use one-year holding-period returns to a 10-year bond are lower than estimates that use 10-year yields. In contrast, the use of rational forecasts of one-year holding-period returns to a 10-year bond in place of 10-year yields will, on average, lead to higher estimates of the cost of equity for low-beta assets. As a theoretical matter, though, it is difficult to justify the use of rational forecasts of one-year holding-period returns to a 10-year bond to measure the risk-free rate. One-year holding-period returns are not risk-free and the Sharpe-Lintner variant of the capital asset pricing model, which the AER employs, requires the use of the return to a risk-free asset.

[39] Mathews, T., *A history of Australian equities*, RBA research discussion paper RDP 2019-04.

A1. Terms of reference

Parties

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Schedule

Date of Agreement	11 August 2021
Term	Start Date:11 August 2021 End Date: 20 August 2021
Fee	
Services	Prepare an independent expert report of 15-25 pages, to be submitted to the Australian Energy Regulator (AER) by Energy Networks Australia that examines the way in which a recent Reserve Bank of Australia (RBA) paper produces estimates of the long-run market risk premium (MRP) for Australia. The report to: » explain why estimates of the MRP that Mathews generates differ from existing estimates; » explain how the series that Mathews constructs differ from the Lamberton, Brailsford, Handley and Maheswaran, NERA and Dimson, Marsh and Staunton series; » explain where Mathews uses methods or techniques not previously used in estimating the MRP for Australia; » explain what impact using arithmetic or geometric mean returns will have on an estimate of the MRP and which measure one should use in estimating the return required on the assets of a regulated energy utility and why; » explain whether the RBA paper has produced better estimates of the MRP for Australia than currently exist; » make clear whether there are problems with the way in which Mathews constructs series of returns and whether these problems, if they exist, will lead the estimates of the MRP that he produces to be upwardly or downwardly biased; and » identify which of the estimates of the long-run MRP for Australia which are available to the AER likely provide the most robust results, taking into account the regulator’s Assessment Principles.

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